Fractional derivative formulae in the form of difference operators

Kuldeep Singh Gehlot and Jyotindra C. Prajapati*
Department of Mathematics,
Government Bangur College, Pali-306401, Rajasthan, India.
E-mail: drksgehlot@rediffmail.com
*Department of Mathematics, Faculty of Technology and Engineering, Marwadi
Education Foundation Group of Institutions, Rajkot-360003, Gujarat, India.
E-mail: jyotindra18@rediffmail.com

Abstract

This paper presents interdisciplinary work between Fractional Calculus and Numerical Analysis. Authors established new formulae of Fractional derivative in the form of Forward and Backward Differences. Fractional derivatives of x^n, $\cos x$ and General Class of polynomial $S_n^m(x)$ with the help of newly defined formulae also obtained.

Key Words: Forward Difference Operator, Backward Difference Operator, Fractional Derivative, Hypergeometric Function.

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1. Introduction

1.1 Notations

Following notations used for deriving several results.

$\triangle_h$ = Forward Difference Operator, $\bigtriangledown_h$ = Backward Difference Operator, D = Differential Operator, E = Shift Operator, I = Identity Operator, h = Interval of Differences, $\mathbb{R}$ = Set of Real Numbers and $\mathbb{N}$ = Set of Natural Numbers.

1.2 Definitions

Let $t \in \mathbb{R}$ and $f(t)$ is a function of t then for $n \in \mathbb{R}$, following Operators defined as:

Shift Operator

$$E^{nh}f(t) = f(t+nh), \ E^{-jh}f(t) = f(t-jh)$$

Forward Difference Operator

$$\triangle_h f(t) = f(t+h) - f(t)$$

Backward Difference Operator

$$\bigtriangledown_h f(t) = f(t) - f(t-h)$$